SIGNAL TO NOISE RATIO - WHAT IS THE RIGHT SIZE?

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ABSTRACT

The concept of signal (S) to noise (N) ratios has high visibility in design of experiment circles due to the work of Taguchi and his disciples. In this paper we will look into one of the signal to noise ratios and provide an answer to the question of "How large the ratio should be to call it a significant signal?"

INTRODUCTION

The first step of this paper will be to define terms that are used in the "Signal to Noise" ratio analyses. The next step will be to select the ratio of interest to this paper. The final step will be to show how, using this ratio, one can arrive an answer to the question posed in the abstract above.

<u>Definition of terms as used in this paper:</u>

Customer Specification Limits are usually stated in terms of three or fewer of the following properties of the product supplied to the customer.

Upper Specification Limit (USL): product exceeding this limit is deemed to be unacceptable by the customer,

Lower Specification Limit (LSL): product falling below this value is deemed to be unacceptable by the customer,

Nominal Value (Nom): this is the value for the property that is most desirable to the customer.

Process Specification Limits at various stages of the manufacturing process (other than the final stage) may be very different from those of Customer Specifications.

DISCUSSION

The terms Signal and Noise, in this paper, are applied to the natural variation of the end product of the process with the Signal being represented by the process average and the Noise being represented by the standard deviation of that output.

There have been a large number of Signal to Noise ratios defined. For definitions of these ratios see, for example, Barker (1990), Devor, Chang and Sunderland (1992), Montgomery (1997). These ratios are commonly used within the context of design of experiments in industry to find the best parameter setting for the process input variables; i.e., the level(s) which will optimize the process output variable. The Signal to Noise (S/N) ratio to be discussed in this paper is the one defined as:

$$\text{S/N} = \frac{\text{Average}}{\text{Standard Deviation}} \tag{1}$$

The type of signal to noise ratio given in equation (1) is just the reciprocal of the coefficient of variation (CV), a statistical measure which is often used in industrial statistics. In the context of experimentation, one would like to be sure that the average value is far enough away from zero that one may assume that it is not simply due to the random variation (i.e. noise).

<u>Questions:</u>

1. What is the physical interpretation of S/N?
2. How large is a large for S/N or conversely, how small is small for CV?

#1

Question #1 may be answered as follows. The S/N ratio is a measure of the magnitude of a data set relative to the standard deviation. If the S/N is large, the magnitude of the signal is large relative the "noise" as measured with the standard deviation. If S/N is large, then the signal is deemed to be significant – not just random variation.

#2

Question #2 deals with the issue of how we know whether we have a signal or just a noise (i.e., random variation). This question has not received much attention in the field. We will show a way of answering this question. The answer can be readily extended by the reader to determining the significance of a value for CV. The most commonly used critical value for CV is, based on rule of thumb (or perhaps rule of ten fingers would be more appropriate) is that the CV should be less than 1/10.

$$S/N = \frac{\overline{X}}{S_x} = \frac{\overline{X} - 0}{S_x} \tag{2}$$

where $\overline{X}$ is the average and S_x is the standard deviation. In other words, S/N ratio is similar to testing the whether of $\overline{X}$ is significantly different than zero. The above ratio gives us a clew as to the answer to the second question – "How large is large?" Since $S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$ where n is the sample size used to obtain the value for $\overline{X}$, we can see that

$$S/N = \frac{\overline{X} - 0}{\sqrt{n}\, S_{\bar{x}}} = \frac{Z(\overline{X})}{\sqrt{n}} \tag{3}$$

And thus

$$(S/N)\sqrt{n} = Z(\overline{X}) \tag{4}$$

Hence if one were to multiply the S/N ratio by the $\sqrt{n}$, one would be testing the hypothesis that the average value of X is significantly different from zero. For example, S/N is considered large (i.e., representing a signal), if $(S/N)\sqrt{n} > 3$, for a confidence level of 99.7%. This implies an existence of a "signal" over and above noise. If $(S/N)\sqrt{n}$ is less than 3, this implies that the data could very well be simply Noise. And inline with the CV expression $CV = \dfrac{1}{S/N}$, one can see that significant values of CV are those below 0.33 for a confidence level of 99.7%.

EXAMPLE

Suppose the data in Table 1 below represents different process experiment conditions, i.e., each row represents a process experiment condition and each column represents a repeat under that condition (Table 1).

Table 1. Eight process experiment conditions with repeat trials.

Process Experiment Conditions	Repeat Trials							
1.	4.62	1.15	0.37	0.23	0.30	0.57	3.99	0.95
2.	5.61	0.45	0.20	0.03	0.47	0.82	1.46	0.43
2.	6.03	0.66	1.07	1.02	1.24	0.17	0.95	0.83
4.	3.66	4.24	3.04	3.93	2.56	4.27	8.01	3.06
5.	0.24	0.16	0.73	0.96	3.63	0.98	0.56	0.86
6.	0.07	0.49	0.79	0.16	4.44	0.88	0.39	0.30
7.	1.45	0.82	0.91	0.92	0.23	0.72	4.72	1.26
8.	0.18	0.22	1.43	0.76	4.02	0.97	0.64	0.98

When we apply the S/N criteria developed above in equation 4 to the data in Table1, we see that all of the process experiment conditions, except the fourth one, do not have significant test statistics

(Table 2). Process condition number four (i.e., row four) has the highest S/N ratio to indicate that it is a real signal in the sense that it is outside the range of variability of the process noise.

Table 2. (S/N)* $\sqrt{8}$ Values for each process experiment condition.

Process Experiment Conditions	(S/N)*√8
1.	2.454292
2.	1.819034
3.	2.274705
4.	6.831187
5.	2.607344
6.	1.843745
7.	2.789658
8.	2.643928

SUMMARY

This paper provides a guideline for deciding whether or not a $\frac{\overline{X}}{S_x}$ type S/N ratio indicates the presence of a meaningful signal. We believe criteria, such as the one discussed in this article, should be used to judge the significance of the ratio rather than relative ranking of the ratios among each other in designing process conditions.

For a benchmark S/N ratio, one could use the capability standard deviation in the denominator, such as standard deviation estimated using mean square successive differences (S_{MSSD}), (see, for example, Holmes and Mergen (1995)) to get the best value of the S/N possible when the process is statistically stable (i.e., displaying only random variation).

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